Answer Key

Pre Calculus 11

Midterm Exam Review Package

The midterm exam will include <u>everything</u> that we have covered in term 3. The topics covered include:

Unit 1: Chapter 3: *Solving Quadratic Equations*

Unit 2: Chapter 4: *Analyzing Quadratic Functions*

Unit 3: Chapter 5: *Graphing Inequalities and Systems of Equations*

Unit 4: Chapter 7 *Rational Expressions and Equations*

This package includes practice questions for all of these topics, as well as some beautiful English vocabulary and key points to remember. Once you complete all of this, try any questions in your textbook that you have not completed! Happy Studying! ☺

Name: Answer Key

PC11 Midterm Review
Midterm Date: TBA

CHAPTER 3

1. Solve the following equations

a. $49x^2 - 121y^2 = 0$

$(7x+11y)(7x-11y)=0$

b. $10x^2 + 23x - 5 = 0$

$(5x-1)(2x+5)=0$

$x_1 = 1/5 \quad x_2 = -5/2$

c. $8x^2 + 22x - 21 = 0$

$(4x-3)(2x+7)=0$

$x_1 = \frac{3}{4} \quad x_2 = -7/2$

d. $9x^2 + 12x + 4 = 0$

$(3x+2)^2=0$

$x = -2/3$

e. $x^2 - 10x + 3 = 0$

use formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_1 = 6.31$

$x_2 = 9.69$

f. $\sqrt{x+1} - 5 = x$

$x+1 = (x+5)^2$

$x^2 + 9x + 24 = 0$

∴ No Solution $(b^2-4ac<0)$

2. Without solving, how many roots do the following equations have:

a. $5x^2 - 8x + 6 = 0$

$b^2-4ac = (-8)^2-4(5)(6)$

$= -56$

$-56 < 0$ ∴ No Real Roots

b. $b^2 - 4ac = 0$

$b^2-4ac = 0$

∴ One Real Root

CHAPTER 4

1. Describe all the transformations

$$y = 4(x-3)^2 + 9$$

Vertical Shift ⇒ 9 units up

Horizontal Shift ⇒ 3 units Right

Vertical Stretch ⇒ by a factor of 4 (upwards)

CHAPTER 5

1. Represent the following inequality on a number line

$$2x^2 - 7x > -3$$

$2x^2-7x+3>0$

Step 1: Find Critical Values

$(2x-1)(x-3)=0$

$x_1=\frac{1}{2}$ $x_2=3$

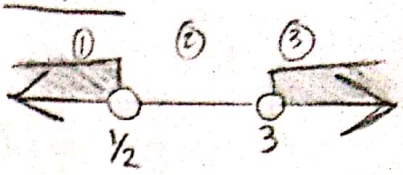
Step 2: Intervals

① $x<\frac{1}{2}$: Choose 0 : $0>-3$ True

② $\frac{1}{2}<x<3$: Choose 1 : $-5>-3$ False

③ $x>3$: Choose 4 : $4>-3$ True

Step 3:



∴ $x<\frac{1}{2}$ or $x>3$

2. Solve the inequality

$$5x \geq 2(x^2 - 6)$$

$0 \geq 2x^2-5x-12$

$0 \geq (2x+3)(x-4)$

$x_1=-\frac{3}{2}$

$x_2=4$

Intervals

① $x \leq -\frac{3}{2}$: Choose -2 : $-10 \geq 4$ False

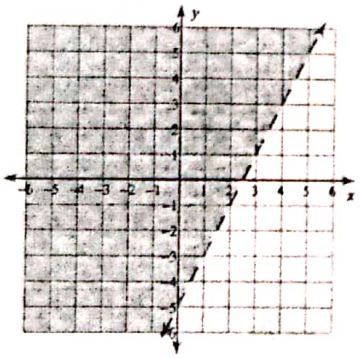
② $-\frac{3}{2} \leq x \leq 4$: Choose 0 : $0 \geq -12$ True

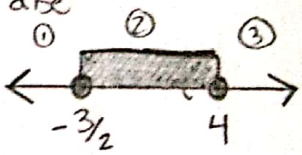
③ $x \geq 4$: Choose 5 : $25 \geq 36$ False

∴ $-\frac{3}{2} \leq x \leq 4$

3. Write an inequality to describe this graph

a.





y-Int: $(0,-5)$

Slope: $\frac{2}{1}$

$y=mx+b$

$y=2x-5$

∴ 2 options

$y>2x-5$ or $y<2x-5$

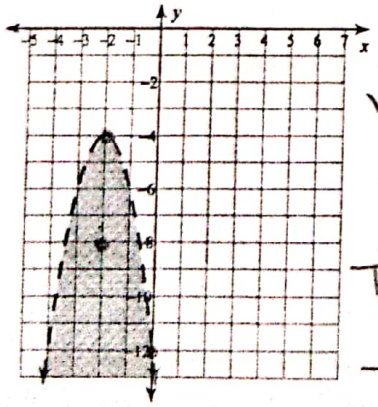
Test point: $(0,1)$

$1<2(0)-5$ $\quad 1>2(0)-5$

$1<-5$ False $\quad 1>-5$ True!

∴ $y>2x-5$

b.



Vertex: $(-2,-4)$

$y=a(x+2)^2-4$

To find "a" use point $(0,-12)$

$-12=a(2)^2-4$

$-8=4a$

$a=-2$

∴ 2 options:

$y<-2(x+2)^2-4$ or $y>-2(x+2)^2-4$

Test Point: $(-2,-8)$

$-8<-4$ True ✓

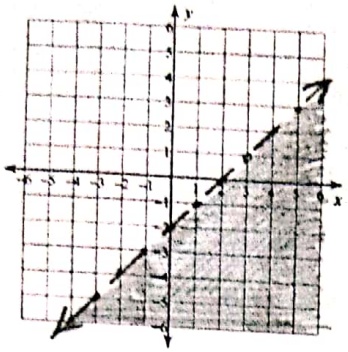
$-8>-4$ False ✗

∴ $y<-2(x+2)^2-4$

4. Draw the inequality

a.

12) $x - y > 2$



$x - y > 2$

$-y > -x + 2$

$\boxed{y < x - 2}$

Test Point: $(1,1)$

$1 < 1 - 2$

$1 < -1$ False

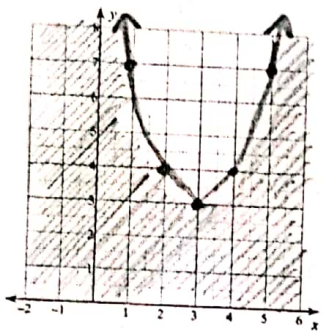
y-Int: $(0,-2)$

Slope: $\frac{1\uparrow}{1\rightarrow}$

$<$ ∴ dashed line - - -

b.

5) $y \leq x^2 - 6x + 12$



$y \leq (x^2 - 6x + (3)^2 - (3)^2) + 12$

$y \leq (x-3)^2 + 3$

Vertex: $(3,3)$

$a = 1$

Test Point: $(3,6)$

$6 \leq (3)^2 - 6(3) + 12$

$6 \leq 3$ False!

5. Solve the system of equation

$y = 3x^2 + 6x + 1$ ①

$2x + y = 12$ ②

Use Substitution:

① → ②

$2x + 3x^2 + 6x + 1 = 12$

$3x^2 + 8x - 11 = 0$

$(3x + 11)(x - 1) = 0$

∴ $\begin{cases} x_1 = -11/3 \\ x_2 = 1 \end{cases}$

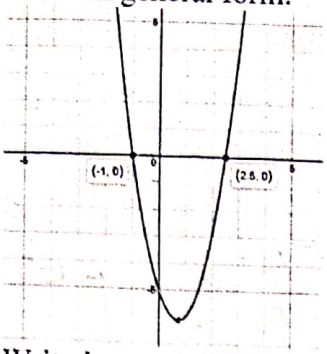
∴ 2 Solutions

$x_1 = -11/3$ $\quad x_2 = 1$

$y_1 = \frac{58}{3}$ $\quad y_2 = 10$

2. Determine an equation for the following graphs

a. Write in general form.



X-Int: $(-1,0), (5/2,0)$ $\quad y=2(x-5/2)(x+1)$

Use Factored Form

$y = a(x-\frac{5}{2})(x+1)$

Use Point: $(0,-5)$

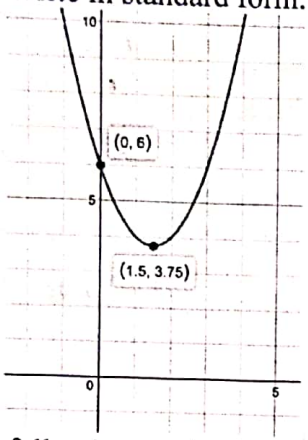
$-5 = a(-5/2)(1)$

$a = 2$

General Form:

$y = 2x^2 - 3x - 5$

b. Write in standard form.



Vertex $(3/2, 15/4)$

$y = a(x-p)^2 + q$

$y = a(x-3/2)^2 + \frac{15}{4}$

Use Point: $(0,6)$

$6 = a(-3/2)^2 + 15/4$

$a = 1$

Standard Form:

$y = (x-\frac{3}{2})^2 + \frac{15}{4}$

3. For the following quadratic function $y = -2x^2 - 6x + 20$

a. Find the vertex

$y = -2x^2 - 6x + 20$

$y = -2(x^2+3) + 20$

$y = -2(x^2 + 3 + (3/2)^2 - (3/2)^2) + 20$

$y = -2(x+3/2)^2 + 20 + 2(3/2)^2$

$y = -2(x+3/2)^2 + 49/2$

Vertex: $(-3/2, 49/2)$

b. Find the zeroes (x-Intercepts)

Let $y=0$

$0 = -2x^2 - 6x + 20$

$0 = x^2 + 3x - 10$

$0 = (x+5)(x-2)$

$\therefore \begin{cases} x_1 = -5 \\ x_2 = 2 \end{cases}$

c. Find the y-intercept Let $x=0$

$y = 20$

$\therefore$ y-Int: $(0, 20)$

d. Find the domain and range

Domain (Possible x-Values) = $x \in \mathbb{R}$

Range (Possible y-values) = $y \leq \frac{49}{2}, y \in \mathbb{R}$

4. What is the axis of symmetry of the following graph? Solve for A.O.S two different ways.

$y = 2x^2 + 6x - 20$

Method 1 - Find Vertex

$y = 2(x^2+3x) - 20$

$y = 2(x^2+3x+(3/2)^2-(3/2)^2) - 20$

$y = 2(x+3/2)^2 - \frac{49}{2}$

Vertex: $(-3/2, \frac{-49}{2})$

∴ A.O.S: $x = -3/2$

Method 2 - use x-Int

$0 = 2x^2+6x-20$

$0 = x^2+3x-10$

$0 = (x+5)(x-2)$

$x_1 = -5$, $x_2 = 2$

A.O.S: $x = \frac{x_1+x_2}{2} = \frac{-5+2}{2}$

∴ A.O.S: $x = -3/2$

5. Word problem

a. Two numbers have a sum of 20. Does the sum of their squares have a maximum or a minimum value? Determine this value and the two numbers

$\begin{cases} x+y=20 \\ x^2+y^2 = \text{Min or Max?} \end{cases}$

$f(x) = x^2+y^2$

$f(x) = x^2+(20-x)^2$

$f(x) = x^2+400-40x+x^2$

$f(x) = 2x^2-40x+400$

$f(x) = 2(x-10)^2+200$

Vertex: $(10, 200)$

x ↑ ↑ Minimum Value

∴ The minimum value is 200. The two numbers are $x=10$ and $y=10$

b. A student parking costs $20. At this price, 150 students will purchase passes. For every $5 increase in price, 20 fewer students will purchase passes

i. What is the price of a parking pass that will maximize the revenue?

$f(x) = (\text{Students})(\text{money})$

$f(x) = (150-20x)(20+5x)$

$f(x) = -100x^2+350x+3000$

$f(x) = -100(x^2-7/2\,x)+3000$

$f(x) = -100(x^2-7/2\,x+(7/4)^2-(7/4)^2)+3000$

$f(x) = -100(x-7/4)^2+3306.25$

Vertex: $(7/4, 3306.25)$

x ↑ ↑ Maximum

∴ Price $= 20+5x$

$= 20+5(7/4)$

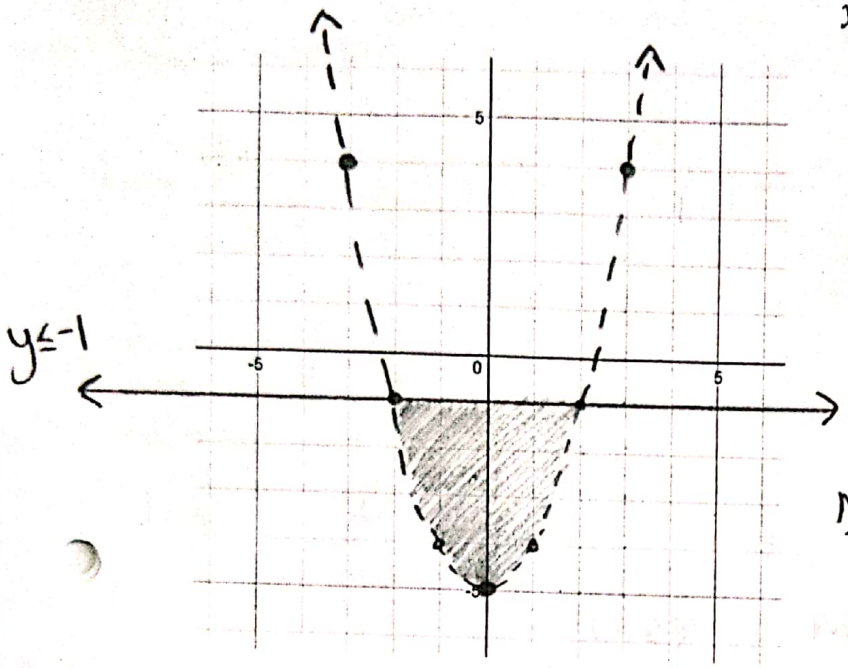
$= \$28.75$

∴ The price to maximize the revenue is $\$28.75$

ii. What is the maximum revenue?

∴ The maximum revenue is $\$3306.25$

6. Draw both inequalities on the same graph. Use the graph to identify all the integer solutions.



$y > x^2 - 5$
$y \leq -1$

$\rightsquigarrow$ $y = (x-0)^2 - 5$

Vertex: $(0, -5)$

$a = 1$

Test Point: $(0, -2)$

$-2 > -5$ True!

Note: only Shade the part which is true for both inequalities!

7. Two numbers are related this way:
① Two times the first number subtract 5 is equal to the second number.
② The sum of the second number and the square of the first number is 115
Create a system of equations to represent this relationship.

① $2x - 5 = y$

② $y + x^2 = 115$

Use Substitution:

① → ②

$2x - 5 + x^2 = 115$

$x^2 + 2x - 120 = 0$

$(x-10)(x+12) = 0$

$\begin{cases} x_1 = 10 \\ x_2 = -12 \end{cases}$

∴ 2 Solutions

$x_1 = 10$	$x_2 = -12$
$y_1 = 15$	$y_2 = -29$

CHAPTER 7

1. Simplify the following expressions and state all the non-permissible values

a. $\dfrac{x^2-9}{12-7x+x^2}$ $x\neq 3$, $x\neq 4$

$= \dfrac{(x-3)(x+3)}{(x-3)(x-4)}$

$= \dfrac{(x+3)}{(x+4)}$

b. $\dfrac{2(x-1)}{3(x-3)} + \dfrac{x-2}{x+2}$

$= \dfrac{2(x-1)}{3(x-3)} \times \dfrac{x+2}{x-2}$

$= \dfrac{2(x-1)(x+2)}{3(x-3)(x-2)}$ $x\neq 2$, $x\neq -2$, $x\neq 3$

$x\neq -10,-5,0$

c. $\dfrac{x+5}{x+10} \div \left(\dfrac{10x}{x^2+10x} \cdot \dfrac{x^2+10x+25}{x^2+15x+50}\right)$

$= \dfrac{x+5}{x+10} \div \left(\dfrac{10x}{x(x+10)} \cdot \dfrac{(x+5)^2}{(x+10)(x+5)}\right)$

$= \dfrac{x+5}{x+10} \cdot \dfrac{(x+10)^2}{10(x+5)}$

$= \dfrac{x+10}{10}$

d. $\dfrac{p-1}{p-2} + \dfrac{p+3}{p+1}$ CD: $(p-2)(p+1)$

$= \dfrac{(p-1)(p+1) + (p+3)(p-2)}{(p-2)(p+1)}$

$= \dfrac{p^2-1+p^2+p-6}{(p-2)(p+1)}$

$= \dfrac{2p^2+p-7}{(p-2)(p+1)}$ $p\neq 2$, $p\neq -1$

e. $\dfrac{n-3}{n^2+3n-18} - \dfrac{n-2}{n^2+n-20}$ $n\neq 3,4,5,-6$

$= \dfrac{(n-3)}{(n+6)(n-3)} - \dfrac{(n-2)}{(n+5)(n-4)}$

$= \dfrac{1}{n+6} - \dfrac{n-2}{(n+5)(n-4)}$

$= \dfrac{(n+5)(n-4) - (n-2)(n+6)}{(n+6)(n+5)(n-4)}$

$= \dfrac{-3n-8}{(n+6)(n+5)(n-4)}$

2. Solve the following rations expressions

$\dfrac{3y-7}{y^2-5y+6} + \dfrac{2y+8}{9-y^2} - \dfrac{y+2}{y^2+y-6} = 0$ CD: $(y-3)(y+3)(y-2)$

$\dfrac{3y-7}{(y-3)(y-2)} - \dfrac{2(y+4)}{(y+3)(y-3)} - \dfrac{y+2}{(y+3)(y-2)} = 0$ $y\neq 2,3,-3$

$(3y-7)(y+3) - 2(y+4)(y-2) - (y+2)(y-3) = 0$

$3y^2+9y-7y-21-2y^2-4y+16-y^2+y+6=0$

$-y+1=0$

$y=1$

3. Word problems

a. A boat travels 4km upstream in the same time that it takes the boat to travel 10km downstream. The average speed of the current is 3km/h. What is the average speed of the boat in still water?

$\therefore t_1 = t_2$

Note: $\text{Speed} = \frac{\text{distance}}{\text{time}}$

$\therefore \text{time} = \frac{\text{distance}}{\text{Speed}}$

Let x = Speed of boat in still water

Time up = Time down

$\frac{\text{distance up}}{\text{Speed up}} = \frac{\text{distance down}}{\text{Speed down}}$

$\frac{4}{x-3} = \frac{10}{x+3}$

$4(x+3) = 10(x-3)$

$4x+12 = 10x-30$

$x = 7$

Note: downstream the boat is faster so the speed of the current (river) must be added

$\therefore$ The Speed of the boat in still water is 7 Km/hr

b. Jenny can clean out the garage in 5hours. When her son helps, they can clean out the garage in 3 hours. How long would it take Jenny's son to clean out the garage on his own?

Let x be time for Son to Clean

$\frac{1}{5} + \frac{1}{x} = \frac{1}{3}$

$\frac{3}{5} + \frac{3}{x} = 1$

CD: $5x$

$\frac{3x}{5x} + \frac{15}{5x} = \frac{5x}{5x}$

$3x + 15 = 5x$

$15 = 2x$

$\therefore x = 15/2$

$\therefore$ It takes Jenny's Son $15/2 = 7.5$ hours to clean the garage.

c. How much bleach should be added to 461 mL of water to make a solution the is 6% bleach?

Note: $6\% = \frac{6}{100}$

Let x be how much bleach to add

CD: $100(x+461)$

$\frac{6}{100} = \frac{x}{461+x}$

$6(x+461) = 100x$

$6x + 2766 = 100x$

$\frac{94x}{94} = \frac{2766}{94}$

$x \approx 29.4$ mL

$\therefore$ 29.4 mL of bleach should be added

Complete all homework in textbook and complete the review questions:

- **Chapter 3 pg. 242-248**
- **Chapter 4 pg. 330-338**
- **Chapter 5 pg. 408-418**
- **Chapter 7 pg. 606- 612**

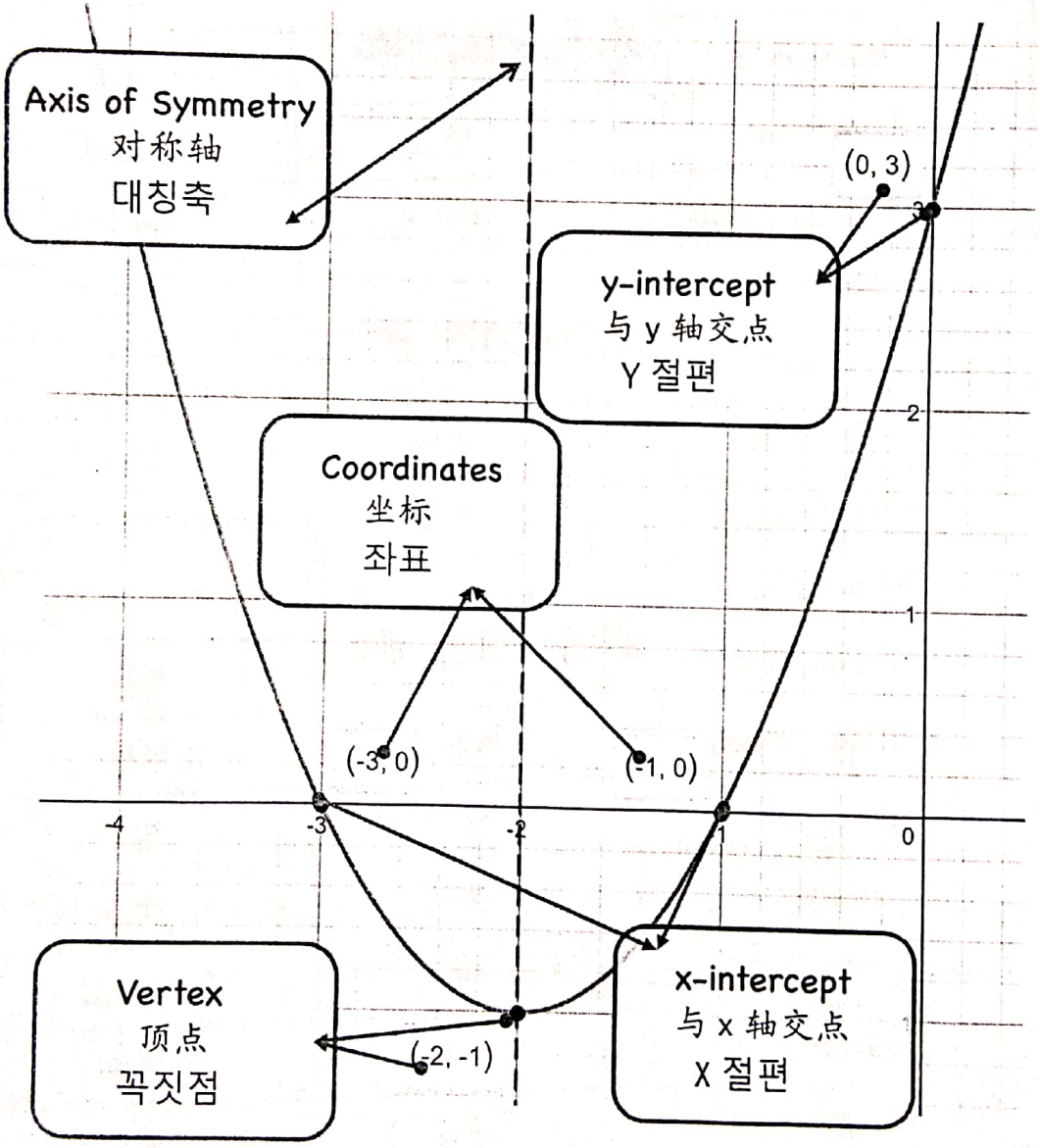
Key Words:

English	Chinese	Korean
Factor	因式/数分解	인수분해
Polynomial	多项式	다항식
Quadratic Equation	二次方程	이차방정식
Solve	解	풀다
Restrictions	限制	불가능한 근
Radical Equation	无理数方程	제곱근
Quadratic Formula	二次函数公式	이차방정식
Nearest Centimeter	答案接近厘米	가장 가까운 cm
Nearest tenth	保留小数点后一位	가장 가까운 소수점 첫 자리
Square Root	平方根	제곱근
Completing the Square	配方法	완전제곱식으로 계산하기
Perfect Square Polynomial	完全平方	완전 제곱식
Roots	根	근
Discriminant	判别式	D 판별식
Table of Values	数据表	값의 표
General Form	一般式	일반형
Parabola	抛物线	이차곡선(포물선)
Vertex	顶点	꼭짓점
Axis of Symmetry	对称轴	대칭축
Domain	X 的取值范围	정의역

English	Chinese	Korean
Range	y 的取值范围	치역
x-intercept	与 x 轴交点	X 절편
Zeros	x 的值	근(x 절편)
Coordinates	坐标	좌표
Sketch	画图	스케치
Horizontal	水平	가로
Vertical	竖直	세로
Standard Form	顶点式	표준형
Minimum	最小	최소값
Maximum	最大	최대값
Stretch	延伸	늘리다
Compression	压缩	수축시키다
Factored form	与 x 轴交点式	인수분해형
Inequality	不等式	부등식
Critical values	临界值	X 절편(근)
Intervals	集合；间隔	범위
Integers	整数	정수
Non-permissible Values	被限制的值	제한값
Variables	变数	변수
Denominator	分母	분모
Numerator	分子	분자

English	Chinese	Korean
Least common multiple	最小公倍数	최소공배수
Simplify	化简	약분
Reciprocal	倒数	음의 역수

Quadratic Function:



General Form
一般式
일반형

$y = ax^2 + bx + c$

Standard Form
顶点式
꼭짓점

$y = a(x - p)^2 + q$

Factor Form
与 x 轴交点式
인수분해형

$y = a(x - x_1)(x - x_2)$

Discriminant
判别式
D 판별식

$b^2 - 4ac$

Numerator	Denominator
分子	分母
분자	분모

$\frac{5}{2}$ $\frac{5}{2}$

Good Luck on your Mid-Term Exam!